# Berlin-Poznań-Hamburg-Warsaw Seminar on Discrete Mathematics 2019

# Berlin-Poznań-Hamburg-Warsaw Seminar on Discrete Mathematics 2019

16 -18 May 2019

Center for Advanced Studies WUT

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# Introduction

The seminar has a long tradition. It was started in mid 1990's by prof. Michał Karoński from the Adam Mickiewicz University in Poznań and prof. Hans Jürgen Prömel (at the time) from Humboldt University in Berlin. The seminar covers a wide variety of topics in extremal and probabilistic combinatorics, algorithmic discrete mathematics, and related fields. In the past the location of the seminar alternated between Berlin, Poznań and Hamburg. This year it is first time organized in Warsaw.

* Talks will be held

at the Faculty of Mathematics and Information Science Main Building, room 103
adress: ul. Koszykowa 75, 00-662 Warszawa

# Programme*

## 17 May 2019 / *Friday*

| | |
|---|---|
| 09:25 - 09:30 | Opening |
| 09:30 - 10:00 | **Simón Piga**, *Localised codegree conditions for tight Hamiltonian cycles in 3-uniform hypergraphs* |
| 10:00 - 10:30 | **Oliver Ebsen**, *Embedding spanning subgraphs in uniformly dense and inseparable graphs* |
| 10:30 - 11:00 | Coffee break |
| 11:00 - 11:45 | **Tomasz Łuczak**, *On Schelp's problem for cycles and paths* |
| 11:45 - 12:15 | **Patrick Morris**, *Tilings in randomly perturbed graphs: bridging the gap between Hajnal-Szemerédi and Johansson-Kahn-Vu* |
| 12:15 - 12:45 | **Joanna Polcyn-Lewandowska**, *The Ramsey-Turán problem for triangles* |
| 12:45 - 14:45 | Lunch break |
| 14:45 - 15:30 | **Jarosław Grytczuk**, *Maps, colors, and polynomials* |
| 15:30 - 16:00 | **Paweł Rzążewski**, *Complexity of $C_{2k+1}$-coloring of graphs with forbidden subgraphs* |
| 16:00 - 16:30 | Coffee break |

| | |
|---|---|
| 16:30 - 17:00 | **Marcin Witkowski**, *Distributed algorithms for minimum dominating set problem* |
| 17:00 - 17:30 | **Simona Boyadzhiyska**, *On counting problems related to (mutually) orthogonal Latin squares* |
| 17:30 - 18:00 | **Alexander Haupt**, *Enumeration of S-omino towers* |
| 18:30 | Dinner |

## 18 May 2019 / *Saturday*

| | |
|---|---|
| 09:15 - 10:00 | **Tamás Mészáros**, *Exploring projective norm graphs* |
| 10:00 - 10:30 | **Zbigniew Lonc**, *Maximin share allocations on cycles* |
| 10:30 - 11:00 | Coffee break |
| 11:00 - 11:30 | **Małgorzata Śleszyńska-Nowak**, *Strong cliques in graphs* |
| 11:30 - 12:00 | **Bjarne Schülke**, *A different Absorbing Lemma for the cycle partitioning problem in r-coloured* $K_n$ |

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# Table of content

**Simón Piga**
Universität Hamburg, Germany

# Localised codegree conditions for tight Hamiltonian cycles in 3-uniform hypergraphs

We study sufficient conditions for the existence of Hamiltonian cycles in uniformly dense 3-uniform hypergraphs. Problems of this type were first considered by Lenz, Mubayi, and Mycroft for loose Hamiltonian cycles and Aigner-Horev and Levy considered it for tight Hamiltonian cycles for a fairly strong notion of uniformly dense hypergraphs. We focus on tight cycles and obtain optimal results for a weaker notion of uniformly dense hypergraphs. We show that if an $n$-vertex 3-uniform hypergraph $H = (V, E)$ has the property that for any set of vertices $X$ and for any collection $P$ of pairs of vertices, the number of hyperedges composed by a pair belonging to $P$ and one vertex from $X$ is at least $(1/4+o(1))|X||P| - o(|V|^3)$ and $H$ has minimum vertex degree at least $\Omega(|V|^2)$, then $H$ contains a tight Hamiltonian cycle. A probabilistic construction shows that the constant $1/4$ is optimal in this context.

**Oliver Ebsen**
Universität Hamburg, Germany

# Embedding spanning subgraphs in uniformly dense and inseparable graphs

We consider sufficient conditions for the existence of $k$-th powers of Hamiltonian cycles in $n$-vertex graphs $G$ with minimum degree $\mu n$ for arbitrarily small $\mu > 0$. About 20 years ago Komlós, Sarközy, and Szemerédi resolved the conjectures of Pósa and Seymour and obtained optimal minimum degree conditions for this problem by showing that $\mu = \frac{k}{k+1}$ suffices for large $n$. Consequently, for smaller values of $\mu$ the given graph $G$ must satisfy additional assumptions. We show that inducing subgraphs of density $d > 0$ on linear subsets of vertices and being inseparable, in the sense that every cut has density at least $\mu > 0$ are sufficient assumptions for this problem. The generalises a recent result of Staden and Treglown.

**Tomasz Łuczak**
Adam Mickiewicz University, Poznań, Poland

# On Schelp's problem for cycles and paths

Following Rado let us write $G \to (H_1, H_2, \ldots, H_k)$ if each coloring of edges of a graph $G$ with $k$ colours leads to a copy of $H_i$ in the $i$-th colours, for some $i = 1, 2, \ldots, k$. About ten years ago Schelp observed that in many cases, if

$$K_n \to (H_1, H_2, \ldots, H_k),$$

then also

$$G \to (H_1, H_2, \ldots, H_k),$$

for each graph $G$ on $n$ vertices with large enough minimum degree. We report on some old and a few new results of this kind, including recent work of Zahra Rahimi and the speaker.

**Patrick Morris**
Freie Universität Berlin, Germany

joint work with **Jie Han** and **Andrew Treglown**

# Tilings in randomly perturbed graphs: bridging the gap between Hajnal--Szemerédi and Johansson-Kahn-Vu

A perfect $K_r$-tiling in a graph $G$ is a collection of vertex-disjoint copies of $K_r$ that together cover all the vertices in $G$. In this paper we consider perfect $K_r$-tilings in the setting of randomly perturbed graphs; a model introduced by Bohman, Frieze and Martin where one starts with a dense graph and then adds m random edges to it. Specifically, given any fixed $0 < \alpha < 1 - 1/r$ we determine how many random edges one must add to an $n$-vertex graph $G$ of minimum degree $\delta(G) \geq \alpha n$ to ensure that, asymptotically almost surely, the resulting graph contains a perfect $K_r$-tiling. As one increases $\alpha$ we demonstrate that the number of random edges required `jumps' at regular intervals, and within these intervals our result is best-possible. This work therefore closes the gap between the seminal work of Johansson, Kahn and Vu (which resolves the purely random case, i.e., $\alpha = 0$) and that of Hajnal and Szemerédi (which demonstrates that for $\alpha \leq 1 - 1/r$ the initial graph already houses the desired perfect $K_r$-tiling).

**Tomasz Łuczak**[1], **Joanna Polcyn**[1], **Christian Reiher**[2]
[1] Adam Mickiewicz University, Poznań, Poland
[2] Universität Hamburg, Germany

# The Ramsey-Turán problem for triangles

An important question in extremal graph theory raised by Vera T. Sós asks to determine for a given integer $t \geq 3$ and a given positive real number the asymptotically supremal edge density $f_t(\delta)$ that an $n$-vertex graph can have provided it contains neither a complete graph $K_t$ nor an independent set of size $\delta n$.

In our work we concentrate on $t = 3$. For $\delta \in [1/2, 1)$ it follows from Turán Theorem (1941) that $f_t(\delta) = 1/2$. On the other side, in 2010 S. Brandt showed that $f_t(\delta) = \delta$ for all $\delta \in (0, 1/3)$.

We conjecture that for all $k \geq 2$ and for all
we have

$$\frac{k}{3k-1} \leq \delta \leq \frac{k-1}{3k-4}$$

$$f_3(\delta) = k(k-1) - 2k(3k-4)\delta + (3k-1)(3k-4)\delta^2.$$

We established this conjecture for $\delta \in [3/8, 1/2]$ and for all $k \geq 4$ and $\delta \in [k/(3k-1), k/(3k-1) + \varepsilon(k)]$.

**Keywords:** Ramsey-Turán theory.

**AMS Subject Classication:** 05C35, 05C55.

**Jarosław Grytczuk**
Warsaw University of Technology, Warsaw, Poland

# Maps, Colors, and Polynomials

The famous Four Color Theorem asserts that every planar graph can be properly colored with at most four colors. The proof is so involved that actually no human can check it without relying on computer verifications. I will present some recent attempts towards a purely mathematical proof of the Four Color Theorem using graph polynomials. Though a final victory is still far ahead, a small light is perhaps flickering in a dark tunnel. For instance, it can be proved by purely algebraic method that from every planar graph one may remove a matching so that the resulting graph is colorable using any lists of four colors assigned arbitrarily to the vertices. It is tempting to conjecture that a similar statement could hold for an independent set of vertices and lists of size three. And if this is too strong, then perhaps some other combination of weakening-strengthening of the statement will eventually strike home.

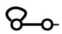
**Paweł Rzążewski**
Warsaw University of Technology, Warsaw, Poland

joint work with **M. Chudnovsky, S. Huang, S. Spirkl** and **M. Zhong**

# Complexity of $C_{2k+1}$-coloring of graphs with forbidden subgraphs

In complexity theory we are often interested in investigating restricted instances of hard problems, in order to understand the boundary between easy and hard cases. A rich family of such restricted instances is given by considering $F$-free graphs, i.e., graphs with no copy of a fixed graph $F$ as an induced subgraph.

Despite a great progress in classifying the complexity of various problems in $F$-free graphs, there are two problems that remain notoriously open: the complexity of 3-Coloring and the complexity of Independent Set in $P_t$-free graphs. Both are known to be polynomially solvable only for some small values of $t$, while no NP-hardness results are known for any $t$.

Some new light on these problems was shed by a recent result of Groenland *et al.* They have proven that if $H$ has no two vertices with two common neighbors, then for every fixed $t$ the weighted variant of $H$-coloring (i.e., a homomorphism to a fixed graph $H$) can be solved in time $2^{O(\sqrt{n})}$ in $P_t$-free graphs. Observe that 3-Coloring is exactly $K_3$-coloring and Independent Set can be expressed as a weighted -coloring. Since none of these target graphs has two vertices with two common neighbors, the result by Groenland *et al.* gives subexponential algorithms for 3-Coloring and Independent Set in $P_t$-free graphs.

In this talk, we investigate another problem of similar flavor – we are interested in deciding homomorphisms to odd cycles. Note that odd cycles do not have two vertices with two common neighbors, so the problem can be solved in subexponential time in $P_t$-free graphs. We show that the problem can be solved in polynomial time in $P_9$-free graphs.

Moreover, we show that the extension version of the problem is NP-complete and cannot be solved in subexponential time in $F$-free graphs, whenever $F$ is not a path or a subdivided claw.

**Marcin Witkowski**
Adam Mickiewicz University, Poznań, Poland

# Finding minimum dominating set in a graph

In graph theory, a dominating set for a graph $G = (V,E)$ is a subset $D$ of $V$ such that every vertex not in $D$ is adjacent to at least one member of $D$. Dominating sets play an important role in many practical applications, e.g. in the context of distributed computing or mobile ad-hoc networks. In the talk I will give a gentle introdution into simple and more advanced distributed algorithms that allows us to very quickly find a set which is close (in size) to minimum dominating set [1, 2, 3]. In general it is NP-hard problem to find such a set in a graph, even when using distributed computing. The problem becomes much more tractable once we restrict the set of graphs to a specific family (e.g. planar graphs, graphs with bounded genus etc.). Methods which are used in the analysis and design of the algorithms comes from the structural graph theory, the talk will focus on the mathematical aspects of the problems.

**References:**

[1] A. Czygrinow, M. Hanckowiak, E. Szymanska, W. Wawrzyniak, M.Witkowski, *Improved distributed local approximation algorithm of the minimum 2-dominating set in planar graphs.* Theoretical Computer Science, Vol 662, (2017), pp.1-8

[2] A. Czygrinow, M. Hanckowiak, W. Wawrzyniak, M. Witkowski, *Distributed algorithms for k-dominating set in bounded genus graphs and linklessly embeddable graphs* 2017 Italian Conference on Theoretical Computer Science (2017)

[3] A. Czygrinow, M. Hanckowiak, W. Wawrzyniak, M. Witkowski, *Distributed CONGESTBC constant approximation of MDS in bounded genus graphs*, Theoretical Computer Science, Vol 757, (2019), pp.1-10

**Simona Boyadzhiyska**
Freie Universität Berlin, Germany

joint work with **Shagnik Das** and **Tibor Szabó**

# On counting problems related to (mutually) orthogonal Latin squares

An $n \times n$ array with entries in $[n]$ such that each integer appears exactly once in every row and every column is called a *Latin square of order* $n$. Two Latin squares $L$ and $L'$ are said to be *orthogonal* if, for all $x, y \in [n]$, there is a unique pair $(i, j)$ such that $L(i, j) = x$ and $L'(i, j) = y$; $k$ Latin squares are *mutually orthogonal* if any two of them are orthogonal.

After the question of existence of a combinatorial structure satisfying given properties, a natural and important problem is to determine how many such objects there are. In this talk, we will consider some counting questions related to (mutually) orthogonal Latin squares. We will present an upper bound on the number of ways to extend a set of $k$ mutually orthogonal Latin squares to a set of $k+1$ mutually orthogonal Latin squares and discuss some applications, comparing the resulting bounds to previously known lower and upper bounds.

**Alexander Haupt**
Technische Universität Hamburg, Germany

# Enumeration of $S$-omino towers

The original problem of counting domino towers was first studied by G. Viennot in 1985, see also D. Zeilberger (*The Amazing $3^n$ Theorem*). We analyse a generalisation of domino towers that was proposed by T. M. Brown (J. Integer Seq. 20.3 (2017), Art. 17.3.1), which we call $S$-omino towers. In the talk we present two arguments for the enumeration of these towers: The first by applying the Lagrange Inversion Formula and another giving an explicit bijection to a certain set of sequences.

**Tamás Mészáros**
Freie Universität Berlin, Germany

joint work with **Tomas Bayer, Lajos Rónyai, Tibor Szabó**

# Exploring projective norm graphs

Let $q$ be a prime power and $t \geq 2$ an integer. The projective norm graph $\mathrm{NG}(q,t)$ has vertex set $\mathbb{F}_{q^{t-1}} \times \mathbb{F}_q^*$ and two vertices $(A,a)$ and $(B,b)$ are adjacent if and only if $\mathrm{N}(A+B) = ab$, where $\mathrm{N}$ stands for the norm function from $\mathbb{F}_{q^{t-1}} \times \mathbb{F}_q$. They were introduced by Alon, Rónyai and Szabó as a tight construction for the Turán problem for complete bipartite graphs $K_{t,s}$ when $s > (t-1)!$ Since their first appearance they have served as useful constructions for other combinatorial problems as well, and many of their interesting properties have been proven.

Here we explore them further and, among others, determine their automorphis groups and study the size of the common neighbourhoods of pairs, triples and quadruples of vertices. The neighbourhood problem is equivalent to determining the number of solutions of some special systems of norm equations.

As a first application we prove that for $t = 4$ the result of Alon, Rónyai and Szabó cannot be strengthened, in the sense that $\mathrm{NG}(q,4)$ already contains many copies of $K_{4,6}$. As another application we count for general $t$ the number of copies of any fixed 3-degenerate graph in $\mathrm{NG}(q,t)$ and find that projective norm graphs are quasirandom with respect to this parameter.

**Keywords:** projective norm graphs, systems of norm equations, quasirandomness.

**Zbigniew Lonc**
Warsaw University of Technology, Warsaw, Poland

joint work with **Mirosław Truszczyński**

# Maximin share allocations on cycles

The problem of fair division of a collection indivisible goods is an important problem in social choice theory.

Consider a finite set of indivisible goods and $n$ agents who have to distribute these goods among themselves. Every good has a value for each agent and each good can be valued differently by different agents. Each agent $x$ imagines the following protocol of division and allocation of goods into agents: $x$ divides the set of goods to $n$ parts, each of the remaining $n-1$ agents chooses one of these parts, and $x$ takes the part that remains. Of course, agent $x$ divides the collection of goods so that he can receive a set of goods of the largest total value regardless of the choices of the other agents. Let us denote this largest value by $m(x)$. The agent $x$ considers the actual allocation of goods to be fair if he gets goods of total value at least $m(x)$. An allocation is called a maximin share allocation, if every agent considers it to be fair.

Now assume that the set of goods has a certain structure. More precisely, the goods are vertices of a certain graph $G$. Not all divisions of the collection of goods between $n$ agents are now allowed, but only those in which each agent gets a set of goods inducing a connected subgraph of $G$. The problem of existence of a maximin share allocation is difficult even for some simple graphs like complete graphs. On the other hand it was shown that maximin share allocations always exist for trees. In the talk we will focus on the case when $G$ is a cycle. Since for cycles maximin share allocations not always exist, we are interested approximating them. More precisely we want to find the largest constant $c < 1$, such that there is always an allocation of a set of goods in which each agent $x$ receives goods of total value at least $c \cdot m(x)$. We prove that, for a cycle, $0.618 < c \leq 0.75$.

**Małgorzata Śleszyńska-Nowak***
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joint work with **Michał Dębski**

# Strong cliques in graphs

Let $G$ be a simple graph. A coloring of the edges of $G$ *is strong* if every path with at most three edges is rainbow. In other words, each color class in a strong coloring forms an induced matching. The least number of colors in a strong coloring of $G$ is called the *strong chromatic index* of $G$, and is denoted by $\chi'_s(G)$. A well-known conjecture of Erdős and Nešetřil from 1985 states that $\chi'_s(G) \leq 1{,}25\Delta^2$ for every graph $G$ with maximum degree $\Delta$.

A *strong clique* in a graph $G$ is a subset of the set of edges whose every pair of elements belongs to a path with two or three edges in $G$. Every usual clique is a strong clique but notice that a complete bipartite graph or a blow-up of 5-cycle are also examples of strong cliques. Let $\omega_s(G)$ denote the maximum size of a strong clique in $G$. Since in a strong coloring of G every strong clique must be rainbow, we have that $\omega_s(G) \leq \chi_s(G)$. So, conjectured upper bound for the strong clique number is $\omega_s(G) \leq 1.25\Delta^2$, while currently best result for the strong chromatic index is $\chi'_s(G) \leq 1.835\Delta^2$.

In our first general result we proved that $\omega_s(G) \leq 1.5\Delta^2$ for any graph with maximum degree $\Delta$. This was recently improved to $\omega_s(G) \leq 1.33\Delta^2$. For some restricted classes of graphs better bounds are possible. For instance, it is known that $\omega_s(G) \leq \Delta^2$ for bipartite graphs, and we proved that $\omega_s(G) \leq \Delta^2 + 0.5\Delta$ for claw-free graphs.

Furthermore, we investigate a more general problem concerning *$t$-strong cliques* in graphs, defined by analogous condition involving paths with up to $t+1$ edges. We prove that the size of a *$t$-strong clique* in a graph with maximum degree is at most $1.75\Delta^t + O(\Delta^{t-1})$, and for bipartite graphs the upper bound is at most $\Delta^t + O(\Delta^{t-1})$. Additional results concern $K_{1,r}$–free graphs and graphs with large girth.

**Bjarne Schülke**
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# A different absorbing lemma for the cycle partitioning problem in the $r$-coloured $K_n$

An important problem in the study of monochromatic partitions is to partition the complete graph $K_n$ for any given (edge) colouring with $r$-colours into as few monochromatic cycles as possible. The currently best known result by Gyárfás, Ruszinkó, Sárközy, and Szemerédi states that $O(r \log r)$ cycles are enough for large $n$. Further, the same authors proved that given a certain kind of partition of almost all vertices into monochromatic cycles, there is a partition of all vertices using at most $3r$ additional cycles.

Now let $m(r)$ be the smallest integer such that for any $r$-colouring of a large almost complete graph, there are at most $m(r)$ disjoint monochromatic connected matchings covering all vertices apart from a small linear sized set. By developing a different Absorbing Lemma and then following a now standard approach due to Łuczak, we show that for large $n$, any $r$-colouring of $K_n$ allows a partition of the vertex set into at most $r + m(r)$ monochromatic cycles. In particular, combined with an earlier result by Gyárfás, Ruszinkó, Sárközy, and Szemerédi, this implies that for large $n$, any 3–colouring of the complete graph $K_n$ allows a partition of the vertex set into 6 monochromatic cycles, improving the currently best known bound of 10 cycles by Lang, Schaudt, and Stein.

**AMS Subject Classification 2010**: Primary: 05C55. Secondary: 05C38

**Keywords:** Edge colourings; Monochromatic partitions